

Multiple removal: an overview and perspective of current capability, algorithmic assumptions and open issues

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June 30, 2021

Introduction

All seismic processing methods make assumptions and have prerequisites. When those assumptions and prerequisites are satisfied the methods can be effective, and problems and challenges arise when those requirements are violated.

This paper can be viewed as the second paper of a two paper set. The first paper asking “why multiples must be removed?” and this paper then asking “how?”

Responding to the inability to provide adequate subsurface information

A critically important assumption made in many current mainstream seismic processing methods is the need for subsurface information.

In the evolution of seismic processing methods, as the algorithms became more effective there was a concomitant increase in the need for more detailed and accurate subsurface information (e.g., for migration, post-stack time, post-stack depth, prestack time to prestack depth) at each step there was a need for increased velocity information.

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That assumption bumped up against the industry trend to deep water and complex offshore and onshore plays. Consequently, the need for adequate subsurface information became increasingly difficult or impossible to satisfy, and that inability remains a major and prioritized challenge to effective seismic processing today. That reality is “the elephant in the room” and is often ignored but is rarely ignorable. That fact drove (and drives) the interest in developing methods that did not need to know, to estimate or to determine subsurface information.

Distinct isolated task subseries of the inverse scattering series were derived and developed for every processing objective. That set of algorithms were (and remain) the only methods that need absolutely no subsurface information to be known, estimated or determined.

Removing the need for subsurface information

An informal history of the development of methods that remove the need for subsurface information for every link in the seismic processing chain can be found in Weglein [2020] and Weglein and Dragoset [2005].

Multiple Removal and Subsurface Information

Multiples are a longstanding problem in seismic exploration.

We catalog multiples as free surface or internal multiples, the former having at least one downward reflection at the free surface, and the latter having all downward reflections beneath the free surface. Multiple attenuation means multiples have been partially removed (amplitude reduced) whereas multiple elimination means the amplitude and phase are predicted exactly at all offsets, and then the subtraction from recorded data removes (i.e., eliminates) the multiple.

The removal and use of multiples have the same exact goal and purpose: the imaging of recorded and unrecorded primaries, respectively. All recorded and unrecorded multiples must be removed to achieve the latter objectives. [Weglein, 2019]

FREE SURFACE MULTIPLES (ELIMINATION)

In Ma et al. [2019] there is a direct comparison of the two leading edge methods for addressing free surface multiples, SRME and ISS FSME (inverse scattering series (ISS) free surface multiple elimination), and when each would be the informed cost-effective choice within the seismic tool box. The different assumptions behind each method are summarized below.

G. Berkhout (1985) and E. Verschuur (1991), pioneered and developed **SRME**, an algorithm that provided an approximate prediction of the phase and amplitude of free surface multiples, without subsurface information, and independent of earth model type. To address the approximate nature of its prediction it called upon an energy minimization adaptive subtraction to remove the multiples.

There are two major sources of algorithmic amplitude and phase errors in SRME:

(1) There is no source and receiver deghosting of the reflection data and the SRME algorithm omits the depth of the sources and receivers beneath the free surface, and

(2) The required obliquity factor, $\sqrt{(\omega/c)^2 - k^2}$ [where ω is the Fourier conjugate to time, t , and k is the Fourier conjugate to the spatial variable, x], is missing in the prediction formula.

The energy minimization adaptive subtraction typically seeks to compensate for errors in the prediction with a temporal frequency dependent function. However, the actual errors from (1) and (2) above are a function of both temporal frequency, ω , and the position of the sources and the receivers. That, in turn, leads to an error (in SRME), that increases with offset and produces a less than effective subtraction (and a residual multiple) at longer offsets. Upon stacking, the multiple will often seem to “reappear” — causing an interpretation challenge. In practice there is often a Radon transform applied, that assumes a 1D CMP moveout pattern and a velocity model. The latter assumption runs at cross purposes with the overall interest in avoiding the need for subsurface information. Those caveats notwithstanding, SRME plus adaptive can be a reasonable choice for removing free surface multiples that are isolated and not proximal to, or interfering with, other events. The principle behind energy minimization can fail with proximal or interfering events.

The inverse scattering series derives a free surface multiple eliminator subseries (ISS FSME) [Carvalho et al., 1992, Weglein et al., 1997, 2003] that inputs source and receiver deghosted reflection data, and naturally incorpo-

rates within its derivation the depth of source and receivers, and the obliquity factor. Hence, it predicts the exact amplitude and phase of all free surface multiples at all offsets, without an energy minimization adaptive subtraction and Radon filtering. In Ma et al. [2019] an example is provided of a free surface multiple that is interfering with a nearby primary. SRME plus adaptive subtraction damages the primary, whereas ISS FSME removes the free surface multiple without damaging the primary. We suggest that ISS FSME be considered as an appropriate tool box choice when free surface multiples might be proximal to or interfering with primaries.

INTERNAL MULTIPLES (ATTENUATION)

For internal multiples the inverse scattering series derived internal multiple attenuator (ISS IMA), Araújo et al. [1994], Weglein et al. [1997, 2003] predicts the exact phase and approximate amplitude of all internal multiples at all offsets. This is a single unchanged algorithm that not only doesn't require any knowledge, estimate or determination of subsurface properties, it is independent of any assumed earth model, (acoustic, elastic, anisotropic, anelastic...). Equally important, the ISS IMA automatically accommodates every possible simple, planar or complex (curved, non-specular, diffractive, pinch-out) multiple generator without any algorithmic change. ISS IMA is the only internal multiple attenuator with that set of properties. For eliminating an isolated internal multiple that is not proximal to (or interfering with) a primary, the ISS IMA will call upon an energy minimization adaptive subtraction to fill the gap between the attenuator amplitude and the amplitude of the internal multiple to be removed.

In Ma et al. [2020] there is a direct field data comparison and analysis of three leading edge internal multiple methods: (1) Jakubowicz [1998], (2) The inverse scattering series internal multiple attenuation (ISS IMA) [Araújo et al., 1994, Weglein et al., 1997] and (3) Marchenko, van der Neut and Wapenaar [2016]. We cite the Ma et al. [2020] paper, for examples, — and provide more detail in this paper on the assumptions behind each of these methods, that explains and supports those test result differences and conclusions. **Marchenko**-based methods towards internal multiple removal have received significant interest in recent years. There are various approaches within the Marchenko portfolio, each with a different set of features and requirements (for subsurface information) that may suit a given multiple contamination

problem, if their assumptions are met.

From two different and important representative Marchenko approaches presented at the recent SEG/KOC Workshop on Multiples Dec. 3-5, 2019, e.g., the one by Wapenaar [2019], involves a full re-datuming of source and receivers as part of the method, and a smooth migration like velocity model is needed. Alternatively, when the method is performed from the seismic experiment recording surface, see, e.g., Dukalski [2019] a virtual boundary defining the multiple generating formations and target ones is required. Although the boundary is virtual, it is required to reside beneath a known and well located physical reflector, requiring prior knowledge and information about the subsurface. This approach and requirement can be interpreted as a form of the Jakubowicz [1998] method and relates to early internal multiple removal concepts by Berkhout. The very close relationship (both in terms of algorithms and assumptions) between the much earlier Berkhout and Jakubowicz approaches and the more recent Marchenko-based methods is detailed and exemplified in Ma et al. [2020], Wu et al. [2020a,b]. For any choice of virtual boundary, certain multiples will be removable and others will not. The paper by Zhan et al. [2019] is an advance within the Marchenko umbrella, that does not require a virtual surface, and showed encouraging results, while pointing out the subsurface and other assumptions made in the method. Among the assumptions is that the data and the multiples are assumed to arise from a reflectivity model of the subsurface, a form of specular reflection model, approximately valid for planar reflectors that will not accommodate curved and diffractive e.g. pinch-out multiple generators. Another assumption is that later-arriving primaries are coming from deeper depths. We know that shallow primary events can have a longer arrival time than a deeper primary, especially at far offset ranges. In such cases, the method assumption is not met, leading to a need of complementary methods to help address the internal multiple contamination issue at far offset. The distinct ISS methods for eliminating free surface multiples, and for attenuating or eliminating internal multiples make none of the assumptions described above for Marchenko methods. In addition, the ISS multiple attenuation methods are able to better resolve and remove multiples from proximal or complex generating reflectors (due to its Stolt Claerbout III water speed migration, Weglein et al., 2016, Zou et al., 2017) compared to, e.g., Jakubowicz [1998] and Marchenko attenuation methods based on intrinsically less capable Kirchhoff or RTM migration.

Ma et al. [2020] conclude that in their field data test and analysis that:

(1) each of these internal multiple methods can provide useful results when their separate and different assumptions are satisfied, and (2) that the most capable of these three methods is the ISS IMA. The ISS IMA provides the exact travel time and approximate amplitude of internal multiples, and is the only method that can automatically accommodate specular and non-specular multiple generators (including pinch-outs) without any subsurface information, and no limiting assumptions about, for example, an assumed relationship that a longer travel time corresponds to a reflector at a deeper depth. The papers by Luo et al. [2011], Wang and Hung [2014], and Ferreira [2011] are a sampling of references that share the same conclusion on the stand-alone capability of ISS IMA. Among other references that contain these same conclusions are: Hung et al. [2014], Matson et al. [1999], Wu and Weglein [2014], Li and Hu [2009], de Melo et al. [2014], Fu et al. [2010]

INTERNAL MULTIPLE (ELIMINATION)

In this section, we focus on a new method that introduces a next generation of needed and necessary internal multiple removal capability.

The approximate amplitude prediction of the inverse scattering series internal multiple attenuator, ISS IMA, requires (in practice) the application of an energy minimization adaptive step to fill the gap between the approximate prediction and actual amplitude of the internal multiple. That can often be an effective strategy, in particular in cases where the internal multiple is not proximal to or interfering with other events.

However, the energy minimum criteria can be invalid when a free surface or internal multiple is proximal to or interfering with other events. The reason is that when a multiple is removed from an interfering primary the “energy” within that spatial and temporal interval can increase, not decrease.

For internal multiples that are proximal to or interfering with other events, a stronger prediction is called for that retains the unique capability and strengths of the ISS attenuator, (ISS IMA) but has the exact time and exact amplitude of the internal multiple, and can remove the internal multiple without calling upon an adaptive subtraction method and step.

That is precisely what the inverse scattering series internal multiple elimination ISS IME responds to and addresses. Zou et al. [2019] provides that next level of capability, with a new concept and algorithm. That paper has a synthetic data test purposefully created with an interfering internal multiple

and a target primary, and a comparison where ISS IMA plus adaptive fails (damaging the target primary) and the new ISS IME removes the internal multiple without damaging the target primary.

The one example included in this paper, is a comparison between the most capable current internal multiple tool box option, the ISS IMA plus adaptive, and the increased internal multiple removal capability of the ISS IME. The latter is an enormously capable and complex and computationally demanding algorithm, far beyond ISS IMA; it was recently pioneered and developed by M-OSRP, and is not yet a tool box option. Hence the single example in this overview paper represents a bridge between the present high water mark internal multiple capability (ISS IMA) and the needed and necessary near future capability (ISS IME). Interfering primaries and internal multiples often occur off-shore and very frequently occur on-shore.

Figures 1, 2, and 3 (from Zou et al., 2019) show a comparison of **ISS IMA** and **ISS IME** for a model where the base salt primary interferes with an internal multiple generated at the water bottom. The **ISS IMA** plus adaptive damaged the base salt primary, whereas the ISS IME removed the internal multiple without damaging the base salt primary.

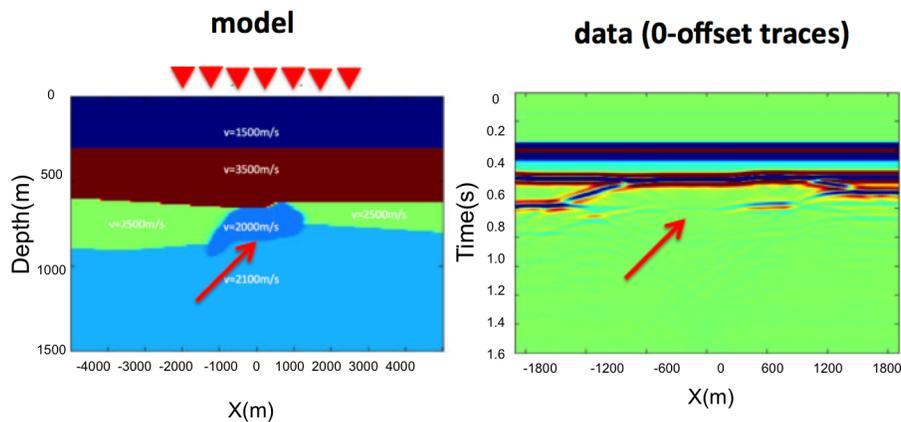


Figure 1: Zero offset traces of data. Note the interfering internal multiple and base salt primary.

The inverse scattering series (ISS) free surface multiple elimination (**ISS FSME**) [Carvalho et al., 1992] algorithm and the ISS internal multiple attenuation (**ISS IMA**) [Weglein et al., 2003] and ISS internal multiple elimi-

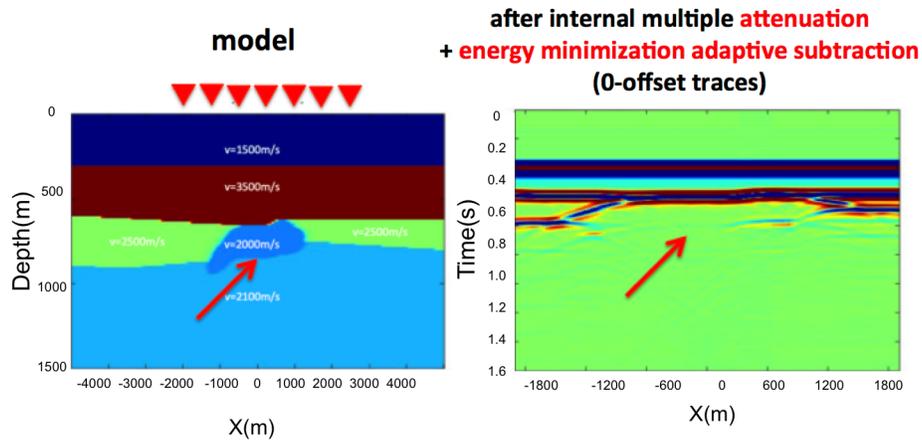


Figure 2: Zero offset traces after ISS internal-multiple attenuation and energy minimization adaptive subtraction. Note the damaged base salt primary.

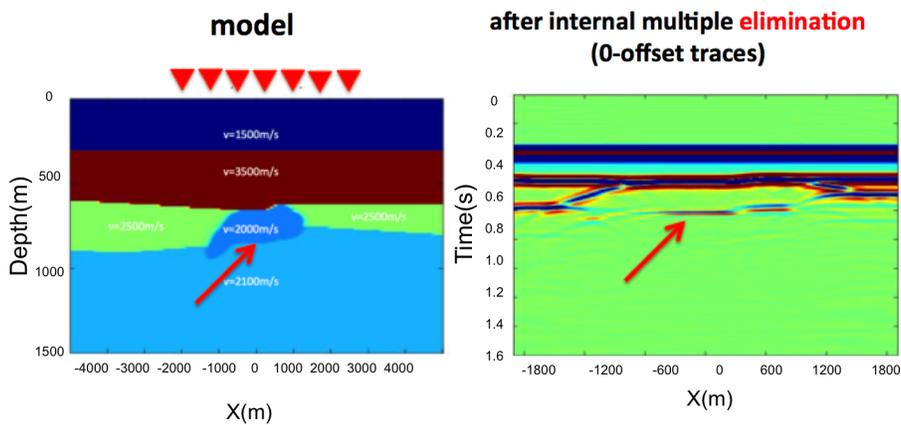


Figure 3: Zero offset traces after ISS internal-multiple elimination. Note the preserved base salt primary.

nation (ISS IME) [Zou et al., 2019] algorithms taken together, represent the high water mark of current multiple removal capability. They remove all multiples, and can automatically accommodate specular and non-specular reflectors, including curved reflectors, diffractive reflectors, and pinch-outs, without (knowing, estimating or determining) any subsurface information, or any knowledge of the generators of the multiples. **They are the only**

methods with that set of capabilities. Furthermore, the ISS IMA and IME algorithms contain a water-speed Stolt-Claerbout III [SCIII] imaging ingredient [and thereby accommodating specular and non-specular reflections, and amplitude information without high frequency approximations] while providing superior illumination and resolution benefits (Weglein et al., 2016 and Zou et al., 2017). The ISS FSME is the method of choice when a free surface multiple is proximal to or interfering with another event, Ma et al. [2019]. In addition, ISS FSME effectively removes free-surface multiples at all offsets, in contrast with SRME.

The comparisons between all current leading edge free surface and internal multiple algorithms (and tool box options) are in the Ma et al. [2019] and Ma et al. [2020] papers, respectively.

On-shore challenges

On-shore exploration presents new daunting challenges. In 2012 Weglein [2013b,a] proposed a three pronged strategy for addressing specific on-shore issues (1) address near surface complexity and surface waves [e.g., separately predicting reflection data and ground roll without filtering and without damaging either] (2) develop on-shore prerequisite satisfaction and multiple elimination methods and (3) find new adaptive criteria that derive as properties of the direct multiple removal methods they are meant to complement and serve.

All current methods for predicting ground roll and reflection data are filtering techniques that remove ground roll while damaging reflection data. The latter is harmful for all subsequent processing goals (e.g., multiple removal, imaging and inversion). Recent significant progress and advances in predicting ground roll and reflection data (without filtering or damaging either), e.g., without needing or determining subsurface properties, but requiring near surface information [Wu and Weglein, 2015, Wu, 2017]. Similarly, Zhang and Weglein [2006] and Matson and Weglein [1996] provide methods for onshore and OBC demultiple and deghosting, respectively, and did not require subsurface information but required near-surface information. New and general methods for seismic preprocessing and processing [Weglein, 2021a], not only do not require subsurface information, but in addition, remove the need for near surface information, as well. The latter is of high priority for onshore plays, and OBS, where the inability to determine near surface properties is one of the most daunting challenges in worldwide petroleum exploration and production. For on-shore (and offshore) applications, Weglein [2012] has proposed an alternate adaptive criteria for free surface multiples that derives as a property of the ISS FSME.

Another important challenge occurs when processing the input events to the internal multiple attenuation and elimination algorithms that are themselves multiples. Liang et al. [2013], Ma and Weglein [2015], Liang [2013], and Ma [2016] provide and illustrate a new ISS method for accommodating primaries and internal multiples in the input data in IMA and IME.

We return to our opening statement that all seismic methods have assumptions and prerequisites. While ISS processing methods have no need to know, estimate or to determine subsurface properties, they (along with all multiple removal methods) do require and assume that the earlier steps in the processing chain be carried out effectively. Those earlier steps include an effective prediction of the reference wave, the reflected wave and the source and receiver deghosted reflection data.

Conclusion

In this paper, we have emphasized and encouraged a tool box perspective, where for each option both the assumptions and advantages have been defined. Recognizing assumptions is important: (1) to communicate what is needed to make the processing method effective, (2) to define the role the method could play in the seismic toolbox, and when it would be the appropriate and cost-effective choice, and when to seek another option and (3) last but not least, to help understand what is behind the breakdown and failure of the algorithms — to help guide research that can produce methods that are effective when all current methods fail. Multiple removal remains (and will remain) a key and central objective in seismic data processing, for as long as we use a smooth velocity model for migration. Please see Weglein [2021b].

For dealing with onshore challenges, Weglein [2013b] proposed a three-pronged strategy. The development of new onshore preprocessing and processing methods that do not need near surface information would be an important step and advance towards realizing a more capable set of on-shore tool box options.

In our view, direct and indirect methods each have a role to play, the former where the assumed physics captures some component of reality and the latter (indirect methods) as the only possible choice for the part of reality that is beyond our physical models, equations and assumptions. Furthermore, it would be ideal if the indirect method and the direct method were cooperative, complementary and consistent. That cooperation can be arranged by choosing

the objective function or sought after quantity to be satisfied (in the indirect solution) as a property of the direct solution (e.g. Weglein, 2012).

In summary, no method is the appropriate and indicated cost effective choice under all circumstances — and for example stacking or Radon filtering could be the method of choice if their assumptions are satisfied in a given circumstance and play. Broadening and increasing the options and collective tool box capability is the goal — and, when needed, to have the option to spend more to deliver more.

We advocate that a research program start by examining the current collective tool box capability, and define what is missing and what new capability would be useful. Then seek to develop a method that adds to that current collective capability. Start with the problem and seek a solution — not with a method seeking a problem.

By understanding the assumptions and prerequisites, we can make an informed cost-effective choice among options. Multiple removal is very far from a closed subject — the research goal is to accommodate a broader set of real world offshore and on-shore circumstances, challenges and plays.

The collective capability within our seismic tool box is always provisional, and it's always a work in progress.

Acknowledgements

A.B.W. and J.D.M. thank the M-OSRP sponsors for their encouragement and support.

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